Quiz #1 (CSE 400.001)

Wednesday, September 11, 2013

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Dept: ____________________ **ID No:** ____________________

1. (7 points) Solve the following initial value problem:

$$\frac{dy}{dx} = \frac{e^{-x}\cos y - e^{2y}\cos x}{2e^{2y}\sin x - e^{-x}\sin y}, \quad y(0) = 0.$$

+1

$$(e^{-x}\cos y - e^{2y}\cos x)dx + (e^{-x}\sin y - 2e^{2y}\sin x)dy = 0$$

$$\frac{\partial M}{\partial y} = -e^{-x}\sin y - 2e^{2y}\cos x = \frac{\partial N}{\partial x} \text{ : exact}$$ +1

$$u = \int (e^{-x}\cos y - e^{2y}\cos x)dx$$ +1

$$= -e^{-x}\cos y - e^{2y}\sin x + l(y)$$

$$\frac{\partial u}{\partial y} = e^{-x}\sin y - 2e^{2y}\sin x + l'(y)$$ +1

$$= N = e^{-x}\sin y - 2e^{2y}\sin x$$

$$\therefore l'(y) = 0, \quad l(y) = C$$ +1

$$u(x,y) = -e^{-x}\cos y - e^{2y}\sin x + C = 0$$ +1

$$u(0,0) = -1 - 0 + C = 0 \Rightarrow C = 1$$

$$\therefore u(x,y) = -e^{-x}\cos y - e^{2y}\sin x + 1 = 0$$

+1

2. (8 points) Solve the following initial value problem:

$$xy' - y + y^2 e^{2x} = 0, \quad y(1) = 2.$$

$$y' - \frac{1}{x}y = -\frac{e^{2x}}{x}y^2 \quad (+1)$$

$$u = y^{-1} \quad (+2)$$

$$u' = (-1)\cdot y^{-2}\cdot y'$$

$$= -y^{-2}\left[\frac{1}{x}y - \frac{e^{2x}}{x}y^2\right]$$

$$= -\frac{1}{x}\cdot u + \frac{e^{2x}}{x} \quad (+2)$$

$$u' + \frac{1}{x}u = \frac{e^{2x}}{x}$$

$$u = e^{-\int \frac{1}{x}dx}\left[\int \left(e^{\int \frac{1}{x}dx}\cdot \frac{e^{2x}}{x}dx\right) + C\right]$$

$$= \frac{1}{x}\cdot\left[\int e^{2x} + c\right]$$

$$= \frac{e^{2x}}{2x} + \frac{c}{x} = \frac{e^{2x}+2C}{2x} \quad (+2)$$

$$y = \frac{1}{u} = \frac{2x}{e^{2x}+2C}$$

$$2 = \frac{2}{e^2+2C} \Rightarrow e^2 + 2C = 1$$

$$\therefore C = \frac{1-e^2}{2}$$

$$\therefore y = \frac{2x}{e^{2x}+1-e^2} \quad (+1)$$