Quiz #2 (CSE 400.001)

Wednesday, September 25, 2013

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1. (8 points) Find a formula involving integrals for a particular solution of the following differential equation:

$$x^3y''' - 3x^2y'' + 6xy' - 6y = f(x), \quad x > 0.$$

$$m(m-1)(m-2) - 3m(m-1) + 6m - 6$$
$$= (m-1)(m-2)(m-3) = 0 \qquad +2$$

$$y_1 = x, \quad y_2 = x^2, \quad y_3 = x^3 \qquad +1$$

$$W = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix} = 2x^3 \qquad +2$$

$$W_1 = x^4, \quad W_2 = -2x^3, \quad W_3 = x^2$$

$$y_p = x\int \frac{x^4}{2x^3}\cdot\frac{f(x)}{x^3}\,dx + x^2\int \frac{-2x^3}{2x^3}\cdot\frac{f(x)}{x^3}\,dx$$
$$+ x^3\int \frac{x^2}{2x^3}\cdot\frac{f(x)}{x^3}\,dx$$
$$= \frac{1}{2}x\int \frac{f(x)}{x^2}\,dx - x^2\int\frac{f(x)}{x^3}\,dx + \frac{1}{2}x^3\int\frac{f(x)}{x^4}\,dx$$

2. (12 points) Solve the following initial value problem:

$$y''' - y'' = (6 + 2x)e^x, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$$

$$\lambda^3 - \lambda^2 = \lambda^2(\lambda - 1) = 0$$

$$y_h = c_1 + c_2 x + c_3 e^x$$

+2

$$y_p = (Ax^2 + Bx)e^x$$

+2

$$y_p' = [Ax^2 + (2A+B)x + B]e^x$$

$$y_p'' = [Ax^2 + (4A+B)x + (2A+2B)]e^x$$

$$y_p''' = [Ax^2 + (6A+B)x + (6A+3B)]e^x$$

+2

$$y_p''' - y_p'' = [(2A)x + (4A+B)]e^x = (2x+6)e^x$$

$$\therefore A = 1, \quad B = 2$$

$$y_p = (x^2 + 2x)e^x$$

+2

$$y = y_h + y_p = c_1 + c_2 x + c_3 e^x + (x^2 + 2x)e^x$$

$$y' = c_2 + c_3 e^x + (x^2 + 4x + 2)e^x$$

$$y'' = c_3 e^x + (x^2 + 6x + 6)e^x$$

+2

$$\begin{cases} c_1 + c_3 = 0 \\ c_2 + c_3 + 2 = 0 \\ c_3 + 6 = 0 \end{cases} \Rightarrow \begin{cases} c_1 = 6 \\ c_2 = 4 \\ c_3 = -6 \end{cases}$$

$$\therefore y = 6 + 4x - 6e^x + (x^2 + 2x)e^x$$

+2