Engineering Mathematics I

Midterm Exam, October 25, 2017

Problem	Score
1	
2	
3	
4	
5	
Total	

Name: ____________________

ID No: ____________________

Dept: ____________________

E-mail: ____________________

1. (20 points) Some diseases are spread largely by *carriers*, individuals who can transmit the disease but who exhibit no overt symptoms. Let x and y, respectively, denote the proportion of susceptibles and carriers in the population. Suppose that carriers are identified and removed from the population at a rate β, so

$$dy/dt = -\beta y. \tag{1}$$

Suppose also that the disease spreads at a rate proportional to the product of x and y, thus

$$dx/dt = -\alpha xy. \tag{2}$$

(a) (5 points) Determine y at any time t by solving Eq (1) subject to the initial condition $y(0) = y_0$.

(b) (10 points) Use the result of part (a) to find x at any time t by solving Eq (2) subject to the initial condition $x(0) = x_0$.

(c) (5 points) Find the proportion of the population that escapes the epidemic by finding the limiting value of x as $t \to \infty$.

2. (20 points) Solve the following initial value problem (without using Laplace transforms):

$$\begin{aligned} y_1' &= 2y_1 \qquad\quad + 2e^{2t}, \quad y_1(0) = 2, \\ y_2' &= 3y_1 + 2y_2 + 3e^{2t}, \quad y_2(0) = 3. \end{aligned}$$

3. (20 points) Solve the following initial value problem by the power series method

$$y'' + x^2 y' + 2xy = 0, \quad y(0) = 1,\ y'(0) = 0.$$

4. (20 points) Using Laplace transforms, solve the following system of differential equations

$$\begin{aligned} y_1' &= 2y_1 \qquad\quad + 2e^{2t}, \quad y_1(0) = 2, \\ y_2' &= 3y_1 + 2y_2 + 3e^{2t}, \quad y_2(0) = 3. \end{aligned}$$

5. (20 points) Prove that $(f * g) * h = f * (g * h)$.