Engineering Mathematics I

Midterm Exam, October 23, 2018

Problem	Score
1	
2	
3	
4	
5	
Total	

Name: _____

ID No: _____

Dept: _____

E-mail: _____

1. (15 points) Suppose that a sum S_0 is invested at an annual rate of return r compounded continuously.

   (a) (10 points) Find the time T required for the original sum to double in value as a function of r.

   (b) (5 points) Find the return rate r if the initial investment is to double in 8 years.

2. (20 points) Solve the following initial value problem

$$y'' - y = 4\sinh x, \quad y(0) = 2,\ y'(0) = 2.$$

3. (20 points) Solve the following initial value problem

$$x^2y'' - 3xy' + 4y = x^2 \ln x, \quad y(1) = 1,\ y'(1) = 0.$$

4. (30 points) Find the Laplace transforms of the following periodic functions

   (a) (15 points) $f(t) = t,\ 0 \le t < 1;\ f(t+1) = f(t)$

   (b) (15 points) $f(t) = \sin t,\ 0 \le t < \pi;\ f(t+\pi) = f(t)$

5. (15 points)

(a) (10 points) If $f(t) = t^m$ and $g(t) = t^n$, where m and n are positive integers, show that

$$f * g = t^{m+n-1} \int_0^1 u^m (1-u)^n du$$

(b) (5 points) Use the convolution theorem to show that

$$\int_0^1 u^m (1-u)^n du = \frac{m!n!}{(m+n+1)!}$$