Name:

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Dept: \_\_\_\_\_ ID No:

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1. (10 points) When  $y_1(x)$  and  $y_2(x)$  form a basis of solutions of the following equation:

$$y''(x) + p(x)y'(x) + q(x)y(x) = 0,$$

show that

$$y_p(x) = -y_1(x) \int \frac{y_2(x)r(x)}{W(x)} dx + y_2(x) \int \frac{y_1(x)r(x)}{W(x)} dx, \text{ with } W(x) = y_1(x)y_2'(x) - y_2(x)y_1'(x),$$

is a particular solution for the following nonhomogeneous linear ODE:

$$y''(x) + p(x)y'(x) + q(x)y(x) = r(x).$$

2. (23 points) Solve the following initial value problem:

$$x^{3}y''' - 2x^{2}y'' - 17xy' - 7y = 45x^{2}\ln x - 24x^{2}, \ (x > 0), \quad y(1) = 1, \ y'(1) = 0, \ y''(1) = 0.$$