$$(A')^T = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ * & d_2 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & * \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 \\ l_{41} & l_{42} & 0 & 1 \end{bmatrix} \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & * & * \end{bmatrix}$$

$$A' = \underbrace{\begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \end{bmatrix}}_{A''} \underbrace{\begin{bmatrix} 1 & l_{21} & l_{31} & l_{41} \\ 0 & 1 & l_{32} & l_{42} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{U}$$

$$Ax = 0 \Leftrightarrow A'x = 0 \Leftrightarrow A''\underbrace{Ux}_{\parallel y} = 0$$

$$(x \neq 0 \Leftrightarrow y \neq 0)$$

$A''y = 0$ for some $y = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \neq 0$

$\Rightarrow Ax = 0$ for some $x = U^{-1}y \neq 0$ #