Geometric Modeling

(CSE 4190.667)

Midterm Exam: November 8, 2017

Problem	Score
1	
2	
3	
4	
5	
Total	

Name: ____________________

ID No: ____________________

1. (20 points) Given a planar cubic Bézier curve with four control points $\mathbf{b}_i = (x_i, y_i)$, $(i = 0, 1, 2, 3)$:

$$C(t) = (x(t), y(t)) = \sum_{i=0}^{3}(x_i, y_i)B_i^3(t), \quad 0 \le t \le 1,$$

represent the following function

$$f(t) = x'(t)y''(t) - x''(t)y'(t) = \sum_{k=0}^{2} f_k B_k^2(t), \quad 0 \le t \le 1,$$

as a quadratic Bézier function by computing the three control coefficients f_0, f_1, f_2 of $f(t)$.

2. (20 points) Let $C(t)$, $(0 \le t \le 1)$, be a cubic Bézier curve given by four control points:

$$\mathbf{b}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \quad \mathbf{b}_3 = \begin{bmatrix} 0 \\ 9 \end{bmatrix}.$$

Subdivide the curve $C(t)$ at x-extreme points, y-extreme points and inflection points, and compute the control points for each inflection-free x, y-monotone curve subsegment.

3. (20 points) Convert the bicubic Bézier patch defined by the following control points to a biquadratic Bézier patch

$$\begin{bmatrix} \begin{bmatrix}0\\0\\0\end{bmatrix} & \begin{bmatrix}48\\0\\0\end{bmatrix} & \begin{bmatrix}60\\0\\0\end{bmatrix} & \begin{bmatrix}36\\0\\0\end{bmatrix} \\ \begin{bmatrix}0\\12\\12\end{bmatrix} & \begin{bmatrix}24\\36\\12\end{bmatrix} & \begin{bmatrix}44\\36\\24\end{bmatrix} & \begin{bmatrix}60\\12\\48\end{bmatrix} \\ \begin{bmatrix}0\\24\\24\end{bmatrix} & \begin{bmatrix}24\\48\\24\end{bmatrix} & \begin{bmatrix}44\\48\\36\end{bmatrix} & \begin{bmatrix}60\\24\\60\end{bmatrix} \\ \begin{bmatrix}0\\36\\36\end{bmatrix} & \begin{bmatrix}48\\36\\36\end{bmatrix} & \begin{bmatrix}60\\36\\36\end{bmatrix} & \begin{bmatrix}36\\36\\36\end{bmatrix} \end{bmatrix}$$

$$\begin{bmatrix} \begin{bmatrix}0\\0\\0\end{bmatrix} & \begin{bmatrix}48\\0\\0\end{bmatrix} & \begin{bmatrix}60\\0\\0\end{bmatrix} & \begin{bmatrix}36\\0\\0\end{bmatrix} \\ \begin{bmatrix}0\\18\\18\end{bmatrix} & \begin{bmatrix}12\\54\\18\end{bmatrix} & \begin{bmatrix}36\\54\\36\end{bmatrix} & \begin{bmatrix}72\\18\\72\end{bmatrix} \\ \begin{bmatrix}0\\36\\36\end{bmatrix} & \begin{bmatrix}48\\36\\36\end{bmatrix} & \begin{bmatrix}60\\36\\36\end{bmatrix} & \begin{bmatrix}36\\36\\36\end{bmatrix} \end{bmatrix}$$

$$\begin{bmatrix} \begin{bmatrix}0\\0\\0\end{bmatrix} & \begin{bmatrix}72\\0\\0\end{bmatrix} & \begin{bmatrix}36\\0\\0\end{bmatrix} \\ \begin{bmatrix}0\\12\\12\end{bmatrix} & \begin{bmatrix}36\\48\\12\end{bmatrix} & \begin{bmatrix}60\\12\\48\end{bmatrix} \\ \begin{bmatrix}0\\24\\24\end{bmatrix} & \begin{bmatrix}36\\60\\24\end{bmatrix} & \begin{bmatrix}60\\24\\60\end{bmatrix} \\ \begin{bmatrix}0\\36\\36\end{bmatrix} & \begin{bmatrix}72\\36\\36\end{bmatrix} & \begin{bmatrix}36\\36\\36\end{bmatrix} \end{bmatrix}$$

$$\begin{bmatrix} \begin{bmatrix}0\\0\\0\end{bmatrix} & \begin{bmatrix}72\\0\\0\end{bmatrix} & \begin{bmatrix}36\\0\\0\end{bmatrix} \\ \begin{bmatrix}0\\18\\18\end{bmatrix} & \begin{bmatrix}18\\72\\18\end{bmatrix} & \begin{bmatrix}72\\18\\72\end{bmatrix} \\ \begin{bmatrix}0\\36\\36\end{bmatrix} & \begin{bmatrix}72\\36\\36\end{bmatrix} & \begin{bmatrix}36\\36\\36\end{bmatrix} \end{bmatrix}$$

4. (20 points) Consider two piecewise cubic Bézier curves with control points $\mathbf{b}_i$, for $i = 0, \cdots, 6$. Let

$$\mathbf{b}_0 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{b}_3 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \mathbf{b}_4 = \begin{bmatrix} 4 \\ 3 \end{bmatrix}.$$

Find the control points $\mathbf{b}_5$ and $\mathbf{b}_6$ so that the piecewise curve is C^3.

5. (20 points) Given a set of data points $\mathbf{p}_k$ and their corresponding parameter values (u_k, v_k):

$$\mathbf{p}_0 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{p}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{p}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{p}_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{p}_4 = \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix},$$

$$(u_0, v_0) = (0, 0),\ (u_1, v_1) = (0, 1),\ (u_2, v_2) = (1, 0),\ (u_3, v_3) = (1, 1),\ (u_4, v_4) = (0.5, 0.5),$$

compute the least squares approximation using a bilinear Bézier surface patch with four control points $\mathbf{b}_{ij}$, $(i, j = 0, 1)$.