Dilation, Erosion, and Outlining

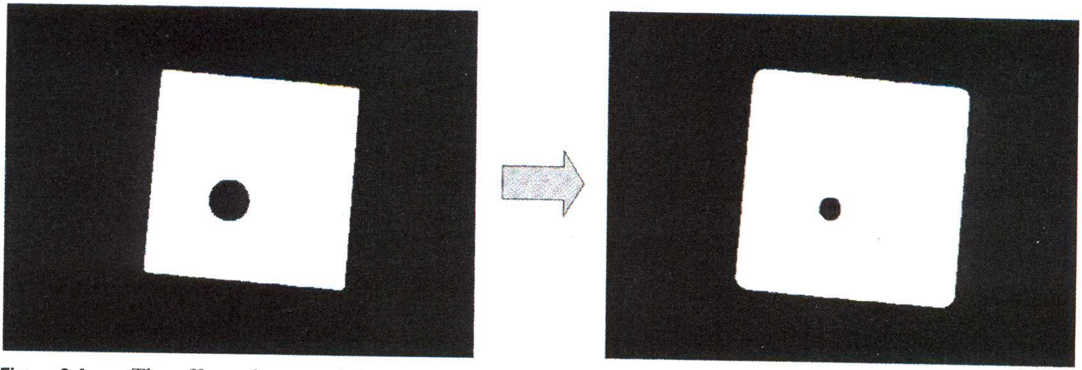


Figure 9.4 The effect of repeated dilation in shrinking background features and smoothing sharp corners, using a structuring element corresponding to 8-connectivity.

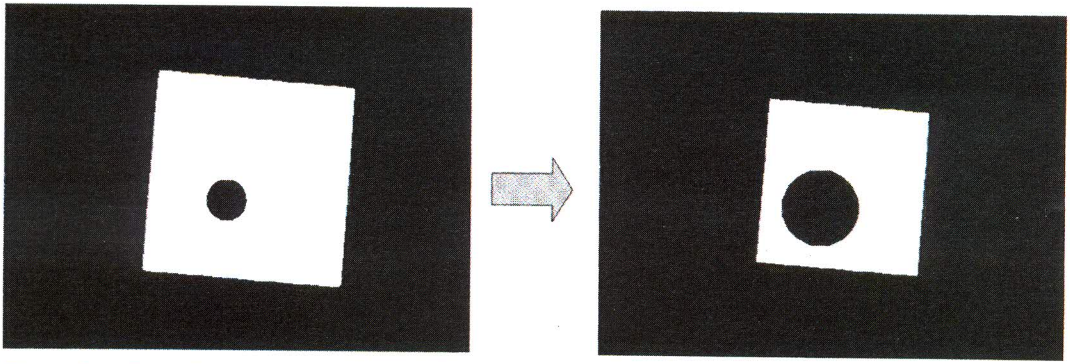
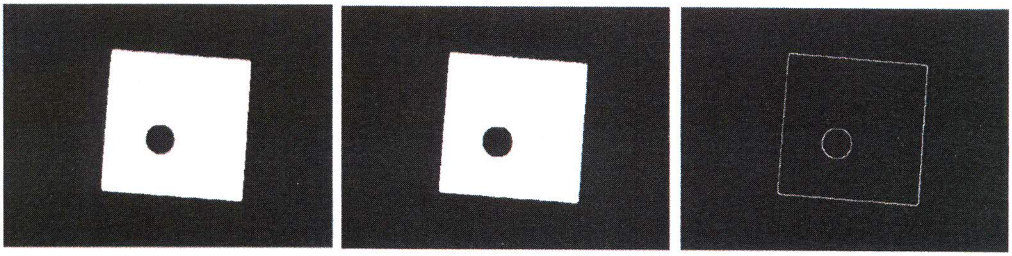


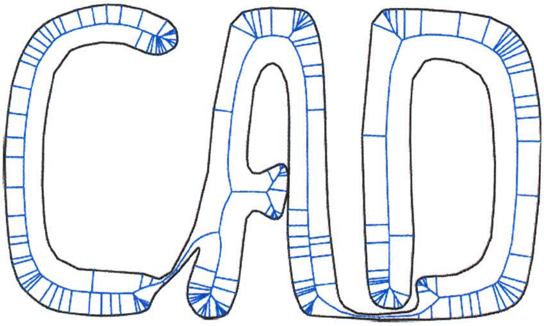
Figure 9.8 The effect of erosion in growing background features and sharpening corners, using a structuring element corresponding to 8-connectivity.



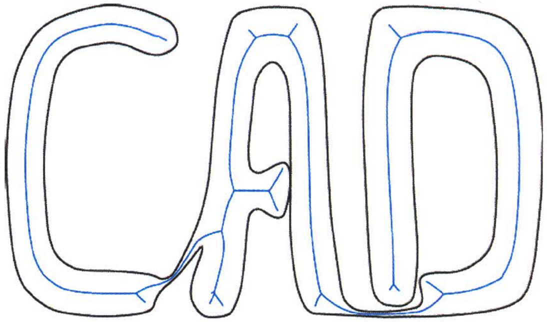
(i) (ii) (iii)

Figure 9.6 Outlining features in an image. (i) Original image; (ii) image dilated (once); (iii) result of subtracting image (i) from image (ii).

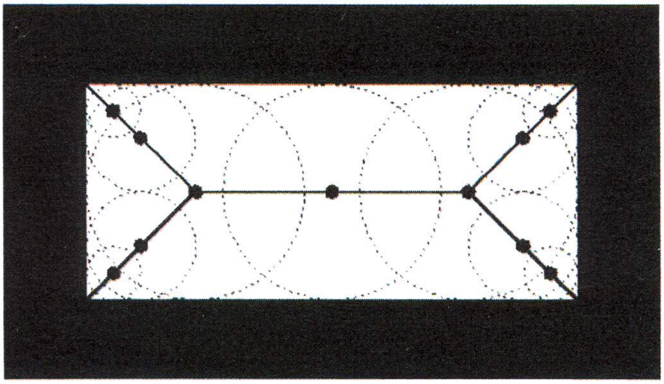
Medial Axis



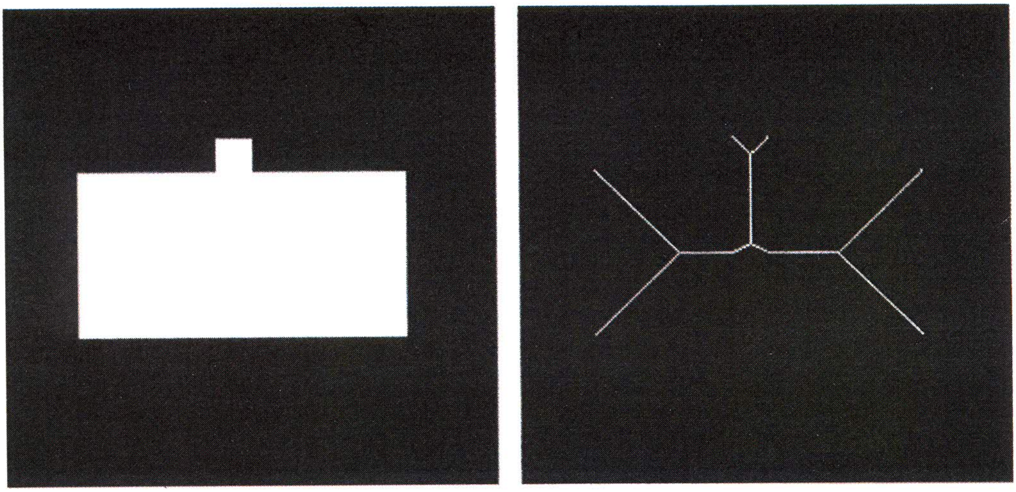
(a) Polygon approximation.



(b) Spiral biarcs approximation.



The skeleton of a rectangle defined in terms of bi-tangent circles.



(i) (ii)

Figure 9.24 (i) An image of a rectangle with a small change in its boundary; (ii) the result of skeletonizing image (i).

Image Skeletonization

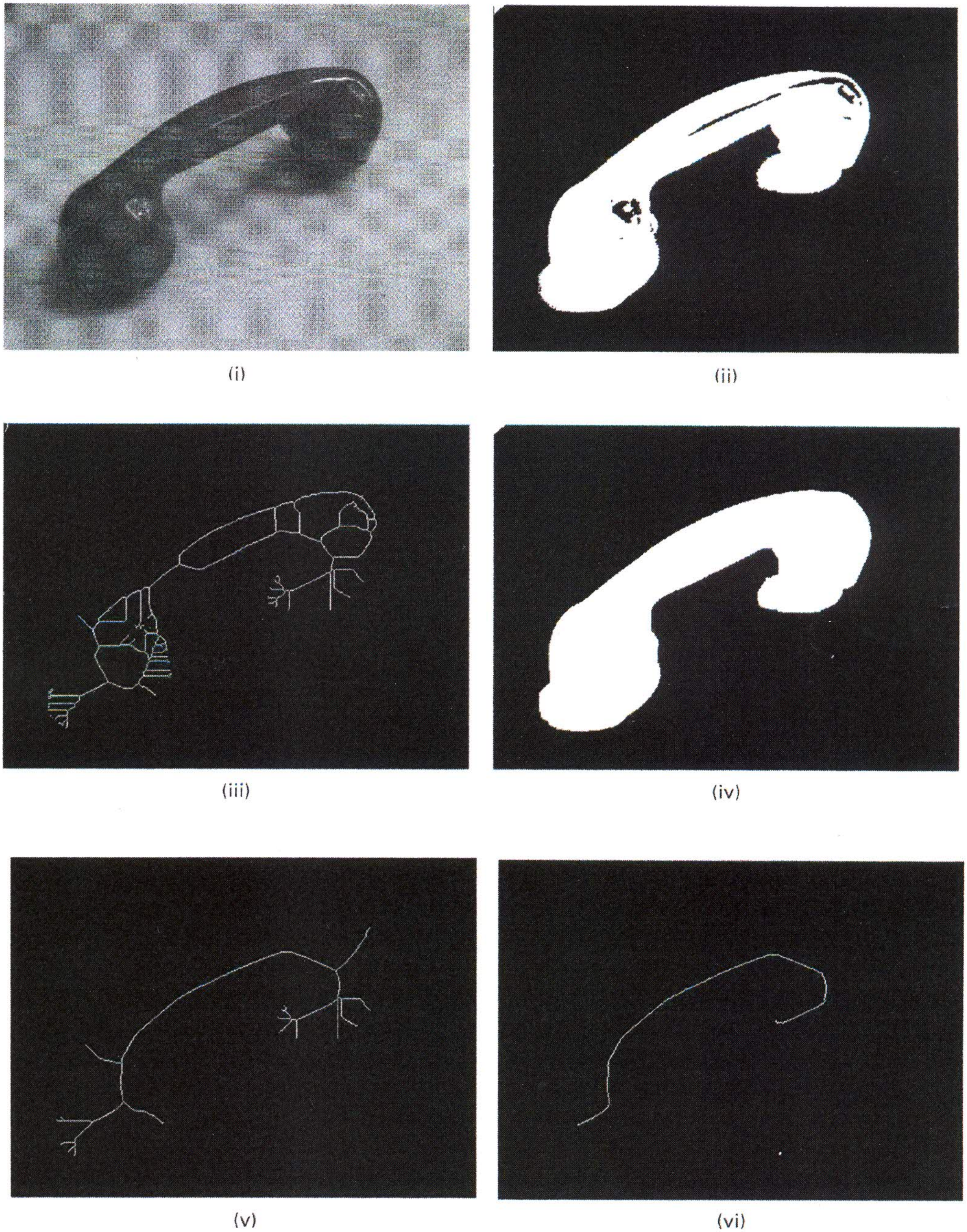


Figure 9.25 (i) Grayscale image of a telephone receiver; (ii) after thresholding image (i); (iii) after skeletonizing image (ii); (iv) after closing image (ii) with a circular structural element; (v) after skeletonizing image (iv); (vi) after pruning image (v).

Convex Hull, Voronoi Diagram, Delaunay Triangulation

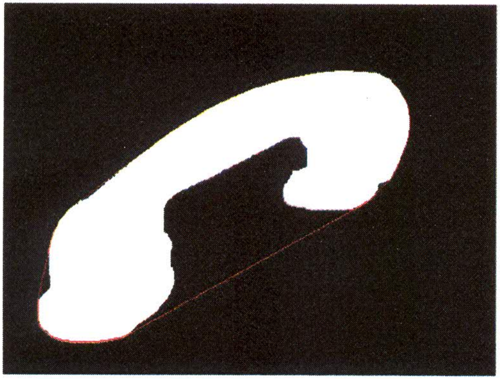


Plate 18 A feature enclosed by its convex hull (shown in red).

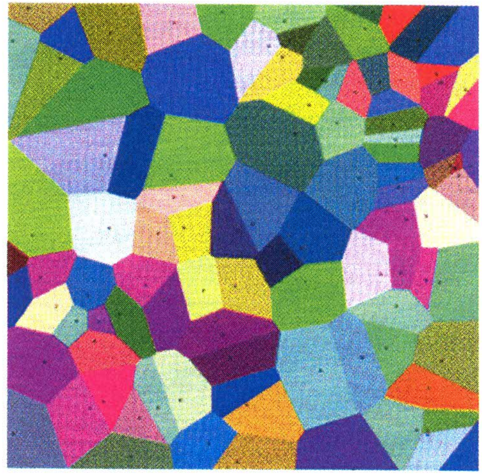
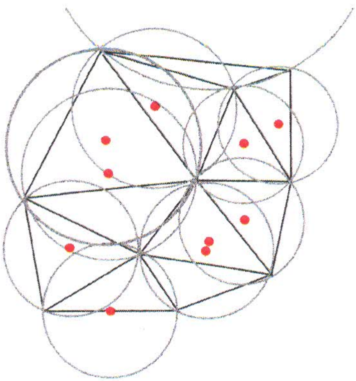
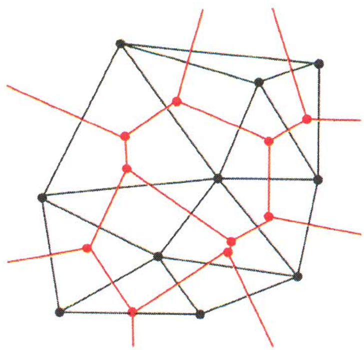


Plate 19 The Voronoi diagram of a set of points, showing the polygons of influence.



(i)

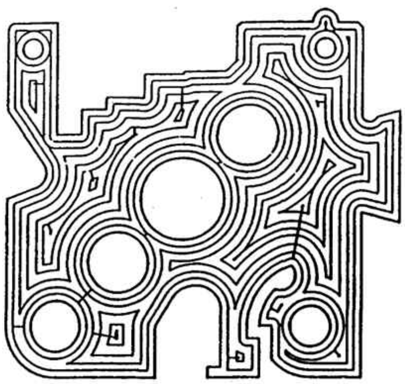


(ii)

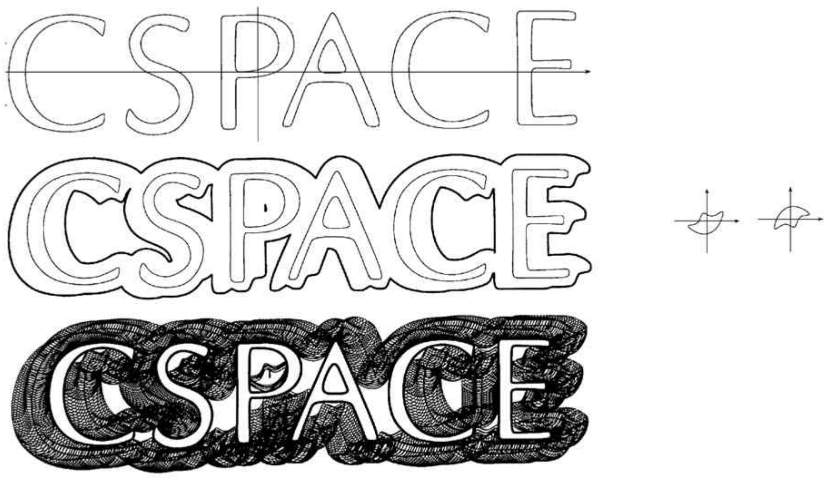
Plate 20 (i) A set of points (in red) with their Delaunay triangulation and circumscribed circles.
(ii) Connecting the centers of the circumscribed points produces the Voronoi diagram (in red).

Offsets, Minkowski Sums, and Sweeps

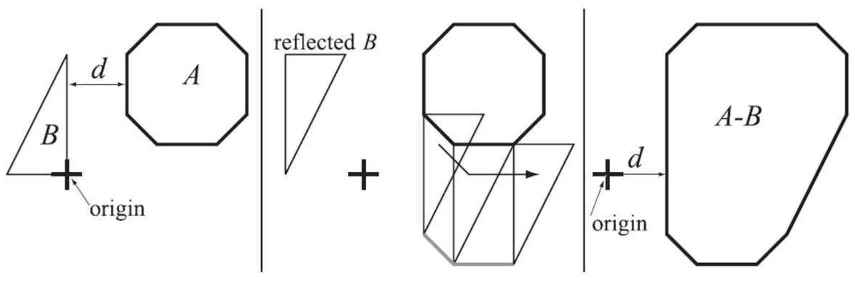
Tool Path Generation for NC Machining



Collision Avoidance Motion Planning

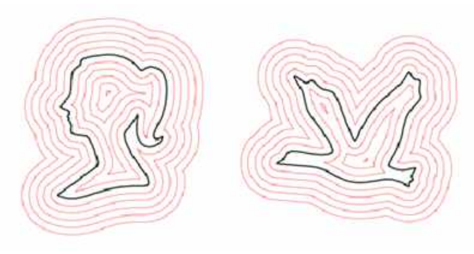


Minimum Distance Computation



$$A - B = \{\mathbf{a} - \mathbf{b} \mid \mathbf{a} \in A, \mathbf{b} \in B\}$$

Planar Offset Curves

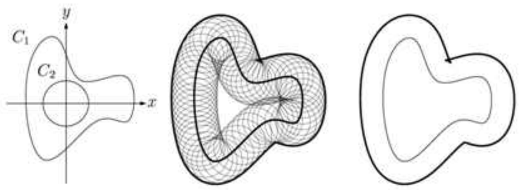


$$C_r(t) = C(t) + r \cdot N(t),$$

$$N(t) = \frac{(y'(t), -x'(t))}{\sqrt{x'(t)^2 + y'(t)^2}}.$$

Offset of A

$$A \uparrow r = \cup_{\mathbf{p} \in A} \; O_r(\mathbf{p})$$

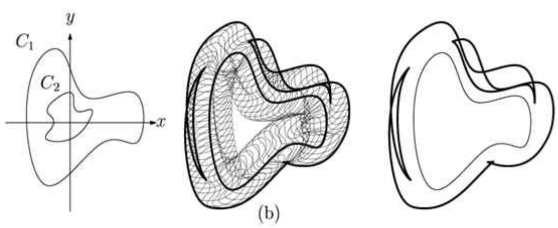


(a)

Minkowski Sum of A and B

$$\begin{aligned} A \oplus B &= \{\mathbf{a} + \mathbf{b} \mid \mathbf{a} \in A, \mathbf{b} \in B\} \\ &= \cup_{\mathbf{a} \in A} \; (B + \mathbf{a}) \\ &= \cup_{\mathbf{b} \in B} \; (A + \mathbf{b}) \end{aligned}$$

$$A \uparrow r \;=\; A \oplus O_r(\mathbf{0})$$



(b)

Envelope Curves

$C_1(u) = (x_1(u), y_1(u))$: Trajectory
$C_2(v) = (x_2(v), y_2(v))$: Moving curve
$(x(u,v), y(u,v))$: Envelope curve defined by

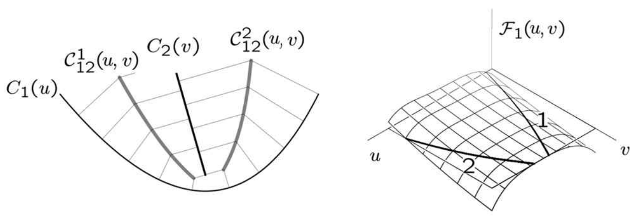
$$\begin{aligned} x(u,v) &= x_1(u) + x_2(v), \\ y(u,v) &= y_1(u) + y_2(v), \\ F(u,v) &= x_1'(u)y_2'(v) - y_1'(u)x_2'(v) = 0. \end{aligned}$$

$$\begin{aligned} x &= x_1(u) + x_2(v) \\ y &= y_1(u) + y_2(v) \\ 0 &= x_1'(u)y_2'(v) - y_1'(u)x_2'(v) \end{aligned}$$

Eliminating u and v, the envelope curve $e(x,y) = 0$ has algebraic degree $O(d^3)$ much higher than $(2d-2)$ of $F(u,v) = 0$.

Bisector Curves

$$\begin{aligned} \Big\langle (x,y) - C_1(u), C_1'(u) \Big\rangle &= 0, \\ \Big\langle (x,y) - C_2(v), C_2'(v) \Big\rangle &= 0, \\ \left\langle (x,y) - \frac{C_1(u) + C_2(v)}{2}, C_1(u) - C_2(v) \right\rangle &= 0. \end{aligned}$$



Eliminating u and v, the curve $b(x,y) = 0$ has degree $7d_1d_2 - 3(d_1 + d_2) + 1$

Eliminating x and y, we have $F(u,v) = 0$ of degree $2(d_1 + d_2) - 2$.

For $d_1 = d_2 = 3$, $F(u,v) = 0$ has degree 10 and $b(x,y) = 0$ has degree 46