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Perspective silhouette of a general swept volume

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G. Elber Dept. of Computer Science, Technion, Haifa, 32000, Israel gershon@cs.technion.ac.il Abstract We present an efficient and robust algorithm for computing the perspective silhouette of the boundary of a general swept volume. We also construct the topology of connected components of the silhouette. At each instant t, a three-dimensional object moving along a trajectory touches the envelope surface of its swept volume along a characteristic curve K^{t} . The same instance of the moving object has a silhouette curve L^t on its own boundary. The intersection $K^t \cap L^t$ contributes to the silhouette of the general swept volume. We reformulate this problem as a system of two polynomial equations in three variables. The connected components of the resulting silhouette curves are constructed by detecting the instances

where the two curves K^t and L^t intersect each other tangentially on the surface of the moving object. We also consider a general case where the eye position changes while moving along a predefined path. The problem is reformulated as a system of two polynomial equations in four variables, where the zero-set is a two-manifold. By analyzing the topology of the zero-set, we achieve an efficient algorithm for generating a continuous animation of perspective silhouettes of a general swept volume.

Keywords Perspective silhouette · Sweep surface · Topology · Zeroset computation · Time varying silhouette

1 Introduction

Silhouettes are among the most important lines in describing the shape of a three-dimensional object. For example, they play a significant role in non-photorealistic rendering [9].

Silhouette curves are dependent on the viewpoint and, in an animation, usually need to be reconstructed for each frame. It is not easy to render silhouettes effectively, as they need to be connected into long smooth strokes if they are to look convincing, and this process has to accommodate complicated topological changes. In this paper, we address this issue by analyzing the topology of the zero-set of a system of polynomial equations. The topological structure of silhouette curves is important not only for correct rendering, but also for analysis of the shape. Elber et al. [6] recently analyzed the topological structure of silhouette curves, and used their analysis to solve the two-piece mold separability problem that occurs in manufacturing processes such as injection molding or die casting. In computer vision, the topology of silhouettes has been utilized in the construction of aspect graphs [4], structures that provide all topologically distinct silhouette configurations.

There has been a lot of research on developing efficient algorithms for computing the silhouettes of polyhedral models (see Isenberg et al. [11] for a recent survey). However, we are concerned with techniques that start with exact models of the shapes involved, and that are of necessity specialized to a particular class of geometries. One

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such class is that of *sweeps*, which are widely accepted as an effective design tool for creating highly complex three-dimensional shapes [1]. Sweeps, whose generating volume changes in size, shape, or orientation as it is swept along a curved trajectory, are called general sweeps [7]. We present an efficient algorithm for computing the silhouette curves of the boundary of a general swept volume.

Joy and Duchaineau [13] have shown how to compute the boundary of a swept volume using a marching cubes algorithm in xyz-space. Kim and Elber [16] reformulated this problem as a polynomial equation in three variables, which is considerably easier to solve. Although they start with exact geometry, these techniques generate a polyhedral approximation of the surface of the swept volume. One may extract the silhouettes from this surface, but the curves need to be approximated by line segments.

Kim and Lee [15] have shown how to compute silhouettes directly without using a polyhedral approximation, but their algorithm is restricted to canal surfaces.

Given a three-dimensional object O, moving under a continuous affine transformation A(t), a swept volume is defined as $\cup_t A(t)[O]$. At a fixed time t, the transformed object A(t)[O] touches the boundary of its swept volume along a characteristic curve K^t . Let L^t denote the silhouette curve of this object A(t)[O] as seen from a viewpoint P (see Fig. 1). The union of the points on the intersection $K^t \cap L^t$ forms the silhouette curves on the boundary of the swept volume.

Tangential intersections between K^t and L^t are critical events in silhouette construction. A new loop of the silhouette curve may start, or an ongoing loop may end, where the two curves K^t and L^t intersect tangentially. We compute all these tangential intersections using three polynomial equations in three variables. The connected components of the silhouette curves are constructed by detecting these critical events and numerically tracing along the common zero-set of the defining equations of the silhouette curves.

Silhouettes change smoothly during an animation, but since they may not have obvious correspondences between frames, it is non-trivial to exploit temporal coherence in the silhouettes when the object is animated [14]. It is also difficult to draw silhouettes smoothly at critical events where the topology of the silhouettes changes. We extend our algorithm to the case of an animation in which the eye position P(r) moves along a predefined path. The problem of computing the critical events then reduces to the solution of four polynomial equations in four variables. Based on the topology information, we extract the silhouette curves at a given eye position P(r) by numerically tracing along the common zero-set of two equations in three variables (r is fixed). To demonstrate the effectiveness of our approach, we present a system that can generate a continuous animation of correctly drawn silhouette curves for a general swept volume.

The rest of this paper is organized as follows. In Sect. 2, we discuss the extraction of silhouette curves; and Sect. 3 deals with their topological structure. In Sect. 4, we consider the topology of time-varying silhouette curves. Experimental results are presented in Sect. 5. Finally, in Sect. 6, we conclude the paper.

2 Extraction of the silhouette curves

We begin by showing how to reduce the problem of computing the perspective silhouette curves of the boundary of a general swept volume to one of solving two polynomial equations in three variables.

Let *O* denote a three-dimensional object bounded by a rational parametric freeform surface S(u, v), and let A(t)denote a continuous affine transformation. The swept volume of the object *O* under the affine transformation A(t)is given as $\cup_t A(t)[O]$. Assuming $a \le t \le b$, the boundary surface of the swept volume consists of some patches of A(a)[S(u, v)] and some of A(b)[S(u, v)], together with



Fig. 1. a When an object *O* moves under a continuous affine transformation A(t), the characteristic curve K^t (in bold lines) touches the boundary envelope surface; **b** the silhouette curve L^t , as seen from the viewpoint **P**, is shown in bold lines and K^t is shown in gray

the boundary envelope surface. The set of points on the envelope surface is characterized by the following equation [16, 17]:

$$F(u, v, t) = \left| A'(t) [S(u, v)] A(t) \left[\frac{\partial S}{\partial u}(u, v) \right] A(t) \left[\frac{\partial S}{\partial v}(u, v) \right] \right|$$

= 0. (1)

That is, the Jacobian of the trivariate volume A(t)[S(u, v)] vanishes on the envelope surface. Since there is only one equation in three variables, the zero-set is a two-manifold in a three-dimensional space.

The silhouette points on the boundary of the swept volume $\bigcup A(t)[S(u, v)]$, seen from a viewpoint **P**, satisfy the following implicit equation:

$$G(u, v, t) = \langle A(t)[S(u, v)] - \boldsymbol{P}, A(t)[N(u, v)] \rangle = 0, \quad (2)$$

where N(u, v) is the normal to S(u, v). Since $N(u, v) = \frac{\partial S}{\partial u} \times \frac{\partial S}{\partial v}$ is rational, the function G(u, v, t) is also rational. The common zero-set of Eqs. 1 and 2 produces 1-manifold curves in *uvt*-space, which correspond to the silhouette curves of the boundary of the swept volume.

Since F(u, v, t) = 0 and G(u, v, t) = 0 are rational equations, their common zero-set can be computed with considerable robustness and reasonable efficiency using the convex hull and subdivision properties of rational spline functions. Solving two equations in three variables, the result is a univariate curve in the *uvt*-space, which can be parameterized by a variable *s*:

(u(s), v(s), t(s)).

See Elber and Kim [5] or Patrikalakis and Maekawa [18] for more details of how to solve a system of *m* polynomial equations in *n* variables.

3 Topology of the silhouette curves

We will now consider how to determine the topological structure of the silhouette curves. For this purpose, we present an algorithm that constructs all the connected components of the silhouette curve.

Consider a point (u, v, t) in the common zero-set of Eqs. 1 and 2. The physical meaning of F(u, v, t) = 0 is that the boundary surface A(t)[S(u, v)] of a moving object A(t)[O] touches the boundary envelope surface of its swept volume $\bigcup_t A(t)[O]$ along a characteristic curve K^t . Further, the condition G(u, v, t) = 0 implies that a surface point A(t)[S(u, v)] is on the silhouette curve L^t , which is itself on the boundary of the moving object A(t)[O]. Figure 1 shows the characteristic curve K^t and the silhouette curve L^t of a moving object A(t)[O]. Under a continuous affine transformation A(t), the intersection points in



Fig. 2. K^t is shown in light gray and L^t is shown in dark gray for $t_{i-1} \le t \le t_i$. The bold black silhouette curve is the union of $K^t \cap L^t$, $\forall t \in [t_{i-1}, t_i]$

the set $K^t \cap L^t$ trace out the whole silhouette curve on the boundary of the swept volume.

Now we consider a connected component $(u(s), v(s), t(s)), (s_0 \le s \le s_1)$, in the common zero-set of F(u, v, t) = G(u, v, t) = 0. Either it forms a closed loop or it has an endpoint at t = 0 and another at t = 1 (Fig. 3). In the case of a closed loop, there are at least two *t*-extreme points (u, v, t) on the loop, which can be computed by solving the following system of three equations in three variables:

$$F(u, v, t) = 0,$$

$$G(u, v, t) = 0,$$

$$H(u, v, t) = F_u G_v - F_v G_u = 0,$$
(3)

where F_u , F_v , G_u and G_v are partial derivatives of F and G. Note that H(u, v, t) is the *t*-component of $\nabla F \times \nabla G$, which is the tangent vector of the zero-set curve. The condition H(u, v, t) = 0 implies that this tangent vector is parallel to the *uv*-plane and thus that the point on the zero-set is a *t*-extreme point. The physical meaning of a *t*-extreme point is that the two curves K^t and L^t touch each other tangentially on the boundary surface of A(t)[O]. (See the two curves at $t = t_{i-1}, t_i$ in Fig. 2.) The other case, in which the endpoints of the curve are at t = 0, 1 can be handled by solving Eqs. 1 and 2 for *u* and *v*.

The simultaneous solutions of Eqs. 1–3 correspond to all *t*-extreme points in the common zero-set of F(u, v, t) = G(u, v, t) = 0, including points that are locally *t*-extreme. Figure 3 shows the three types of connected components encountered in the common zero-set. We characterize three different types of connected components:

- A component of *type 1* is a closed loop (Fig. 3(a)).
- A component of *type 2* has some local *t*-extreme points (Fig. 3(b)).
- A component of *type 3* has no local *t*-extreme point. It is *t*-monotone and its ends are at t = 0, 1 (Fig. 3(c)).



Fig. 3a-c. Classification into three types identifies the topology of a silhouette curve: a a loop; b a curve with local extrema; and **c** a curve that is *t*-monotone. The outlined box represents the domain of the parameter space

Connected components are constructed by numerically tracing the intersection curve F(u, v, t) = G(u, v, t) = 0, starting from the *t*-extreme points or from the endpoints at which t = 0, 1. Algorithm 1 summarizes the whole procedure of constructing the silhouette curve.

Algorithm 1

Input:

S(u, v), a rational freeform surface; A(t), an affine transformation matrix; **P**, the eye position; **Output:** A set of perspective silhouette curves of the boundary of $\bigcup A(t)[S(u, v)]$, as seen from **P**; Begin $F(u, v, t) \Leftarrow |A'(t)[S(u, v)] A(t) \left[\frac{\partial S}{\partial u}(u, v)\right] A(t) \left[\frac{\partial S}{\partial v}(u, v)\right];$ $G(u, v, t) \Leftarrow \langle A(t)[S(u, v)] - \mathbf{P}, A(t)[N(u, v)] \rangle;$ $H(u, v, t) \Leftarrow F_u G_v - F_v G_u;$ $Z_0 \leftarrow$ the common zero-set of *F*, *G*, and *H*; $Z_1 \Leftarrow$ the common zero-set of *F* and *G*; for each solution point $p \in \mathbb{Z}_0$ do Numerically trace a connected component of Z_1 and classify its type;

Parameterize the component according to its type; end

The other solution points of Z_1 are of type 3;

Numerically trace each component starting from the solutions of F(u, v, t) = G(u, v, t) = 0 where t = 0, 1; return a set of silhouette curves;

End.

4 Topology of a time-varying silhouette

Our algorithm can easily be extended to a more general case where the eye position P(r) moves along a predefined path. In this case, Eqs. 1 and 2 are extended to four variables:

$$F(u, v, t) = \left| A'(t)[S(u, v)] A(t) \left[\frac{\partial S}{\partial u}(u, v) \right] A(t) \left[\frac{\partial S}{\partial v}(u, v) \right] \right|$$

= 0, (4)
$$G(u, v, t, r) = \langle A(t)[S(u, v)] - P(r), A(t)[N(u, v)] \rangle = 0.$$

The simultaneous solution to these equations produces a two-manifold zero-set in a four-dimensional space. We now consider how to analyze the topological structure of the time-varying silhouettes. Based on the topology information, we will produce smoothly drawn silhouettes of the swept volume using a non-photorealistic shading model.

4.1 Topology analysis

The *r*-extreme points on the two-manifold surface characterize critical events where a silhouette component may appear or disappear depending on the new eye position P(r). The *r*-extreme point occurs where the gradient vector $\nabla G(u, v, r, t)$ is parallel to the r-direction in uvtrspace. (The partial derivatives G_u , G_v and G_t must vanish at these critical points.) The equation F(u, v, t) = 0 is independent of the time-varying eye position P(r). Thus, the zero-set of F(u, v, t) = 0 forms a cylindrical hyper-surface in *uvtr*-space. We compute the *r*-extreme points on the two-manifold by solving

$$G(u, v, r, t) = 0,$$
 (6)

 $G_u(u, v, r, t) = 0,$ (7)

$$G_v(u, v, r, t) = 0,$$
 (8)

$$G_t(u, v, r, t) = 0.$$
 (9)

Since we have four equations in four variables, their simultaneous solution is a set of discrete points. In the next step, we check whether each discrete solution point also satisfies Eq. 4.

A silhouette component may appear or disappear at the boundary point where u = 0, 1, v = 0, 1, or t = 0, 1. Silhouette components of these types may not have extreme points in the *r*-direction. The end points of these silhouette components are computed by solving Eqs. 6-9. For example, for u = 0, we solve

$$G(0, v, r, t) = 0,$$

$$G_v(0, v, r, t) = 0,$$

$$G_t(0, v, r, t) = 0.$$

(5)

4.2 The silhouette drawing system

To find the silhouettes at a specific eye position P(r) and to render the silhouette strokes during an animation, we trace the two-manifold zero-set surface numerically using the topology information.

- Extraction. A numerical tracing technique extracts an iso-curve of the two-manifold for a fixed value of r = r_0 , which corresponds to a given eye position $P(r_0)$. Since the two-manifold is defined by two implicit equations, $F(u, v, t) = G(u, v, r_0, t) = 0$, where $r = r_0$ is fixed, we can apply a general SSI technique [2, 3] to the tracing algorithm.



Fig. 4. a The envelope of a scaled ellipsoid moving along a linear trajectory, and b its perspective silhouette curves. In c, the silhouette curves are shown in vt-space; two curve components of type 2 were detected

 Drawing. A silhouette curve may be rendered with multiple strokes, for example to produce a dashed line. We construct a single stroke as a smooth curve, and apply it to a periodic texture map suggesting one or more marks. The texture coordinate along the strip is given by the parameterization of the silhouette curve. To illustrate non-photorealistic effects in the animation we use an illumination model similar to Gooch's tone shading [10].

We now present examples of silhouette curves computed

on the boundary of a general swept volume. Figure 4

5 Experimental results

shows the swept volume of an ellipsoid moving along a linear trajectory under a scale change. Its perspective silhouette curves, which are both of type 2, are shown in bold lines in Fig. 4(b). Figure 4(c) shows the projection of the zero-set onto the vt-plane. Two more examples are shown in Fig. 5. These silhouette curves are used in the non-photorealistic rendering of the boundary envelope surfaces of swept volumes.

Figure 6 shows snapshots from an animation of silhouettes in which their topological arrangements change as the eye position moves along a predefined path. At each frame of the animation, we extract a set of connected silhouette components and draw them in bold lines. In a preprocessing step, we detect critical events, in which connected components of the silhouette curves may appear or disappear, by solving a system of polynomial equations.



Fig. 5. a The envelope surface and its silhouette curves (shown in bold lines) for the swept volume of an ellipsoid moving along a trajectory with a scale change. b A tuba is modeled by sweeping a sphere and a torus



Fig. 6a–h. Snapshots of time-varying silhouettes. **a** A swept volume and its perspective silhouettes from an initial eye position; **b–c** transition in which two connected components cross each other; **d** one component disappears in the middle of the object; **e** a new component appears in the middle of the object; **f–g** transition in which two connected components cross each other



Fig.7. a Stylized perspective silhouettes produced using a thorn-like pattern, and b anothor example of stylized silhouette curves with a wave-like pattern

In between two consecutive critical events, the silhouette tracing is stable and efficient.

with a 1GB main memory. Our system generates an animation of silhouettes at about 70 frames per second.

Two connected components of the silhouette curves cross each other in the transition from Fig. 6(b) to Fig. 6(c), and similarly between Fig. 6(f) and Fig. 6(g). One component disappears while moving from Fig. 6(c)to Fig. 6(d). However, a new component appears in the middle of the transition from Fig. 6(d) to Fig. 6(e). In a preprocessing step, we computed the critical events, which took about a second on a P4-2GHz Windows OS

Figure 7(a) shows the result of drawing the silhouette curves of a torus-shaped swept volume in a stylized, non-photorealistic way. A thorn-like pattern was used for texturing the silhouette curves in this example. Figure 7(b) shows the result for a more complicated swept volume. A wave-like pattern was used for texturing the silhouette curves.

6 Conclusion

We have presented a new algorithm for computing the perspective silhouette curves on the boundary envelope surface of a general swept volume without using a polygonal approximation. The silhouette computation was reduced to finding the zero-set of a system of two polynomial equations in three variables. The connected components of the silhouette curves are then detected and constructed using the *t*-extreme points of the zero-set, which we obtain by solving three polynomial equations in three variables. Then the silhouette curves are generated by numerically tracing the zero-set of two polynomial equations. We have further shown that a similar computational paradigm can be applied to a more general case in which the eye position moves along a predefined path. Using this result, we have generated a smooth animation of the perspective silhouettes of a swept volume while the eye position moves around the object.

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References

- Alias-Wavefront Technology. Maya 5.0 Users Manual . http://www.alias.com (2003)
- Bajaj, C., Hoffmann, C., Lynch, R., Hopcroft, J.: Tracing surface intersections. Comput. Aid. Geomet. Des. 5(4), 309–321 (1988)
- Bajaj, C., Xu, G.: NURBS approximation of surface-surface intersection curves. Adv. Comput. Math. 2(1), 1–21 (1994)
- Cipolla, R., Giblin, P.: Visual Motion of Curves and Surfaces. Cambridge University Press (2000)
- Elber, G., Kim, M.-S.: Geometric constraint solver using multivariate rational spline functions. Proc. ACM Symposium on Solid Modeling and Applications, Ann Arbor, MI, June 4-8 (2001)
- Elber, G., Chen, X., Cohen, E.: Mold accessibility via Gauss map analysis. Proc. Shape Modeling International '04, Genova, Italy, pp. 263–272, June (2004)
- Foley, J., van Dam, A., Feiner, S., Hughes, J.: Computer Graphics: Principles

and Practice. 2nd ed, Addison Wesley, Reading, MA (1990)

- FLTK, Fast Light Tool Kit. http://www.fltk.org/. Version 1.1, 2002
- D. Gooch, B., Gooch, A.: Non-Photorealistic Rendering. A.K. Peters. ISBN: 1-56881-133-0 (2001)
- Gooch, A., Gooch, B., Shirley, P., Cohen, E.: A non-photorealistic lighting model for automatic technical illustration. SIGGRAPH'98, pp. 447–452 (1998)
- Isenberg, T., Freudenberg, B., Halper, N., Schlechtweg, S., Strothotte, T.: A developer's guide to silhouette algorithms for polygonal models. IEEE Comput. Graph. Appl. 23(4), 28–37 (2003)
- IRIT 9.0 User's Manual, Technion, October (2002) http://www.cs.technion.ac.il/~irit
- Joy, K., Duchaineau, M.: Boundary determination for trivariate solid. Proc. of Pacific Graphics 99, Seoul, Korea, pp. 82–91, October 5–7 (1999)
- 14. Kalnins, R.D., Davidson, P.L., Markosian, L., Finkelstein, A.: Coherent stylized

silhouette. ACM Trans. Graph. 22(3), 856–861 (2003)

- Kim, K.-J., Lee, I.-K.: The perspective silhouette of a canal surface. Comput. Graph. Forum 22(1), 15–22 (2003)
- Kim, M.-S., Elber, G.: Problem reduction to parameter space. In: Cipolla, R., Martin, R. (eds) The Mathematics of Surfaces IX (Proc. of the 9th IMA Conference), pp. 82–98. Springer, London (2000)
- Martin, R., Stephenson, P.: Sweeping of three-dimensional objects. Comput. Aid. Des. 22(4), 223–234 (1990)
- Patrikalakis, N., Maekawa, T.: Shape Interrogation for Computer Aided Design and Manufacturing. Springer, Berlin Heidelberg New York (2002)
- 19. VTK, The Visualization Tool Kit. http://www.vtk.org/. Version 4.2 (2003)

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